**Exercise 1 - Automobile**

Linearize the momentum balance for the velocity of an automobile at steady state conditions when the gas pedal is maintained at 40%.

With:

* *u(t)* as gas pedal position (%pedal)
* *v(t)* as velocity (m/s)
* *m* as the mass of the vehicle (500 kg) plus the mass of the passengers and cargo (200 kg)
* *Cd* as the drag coefficient (0.24)
* ρ as the air density (1.225 kg/m3)
* *A* as the vehicle cross-sectional area (5 m2)
* *Fp* as the thrust parameter (30 N/%pedal).

**Solution**

1. Determine the steady state value for v.

At steady state, , so:

1. Linearize

* The constant α is the partial derivative of f(v,u) with respect to v and evaluated at steady state conditions.
* The constant is the partial derivative of f(v,u) with respect to u and evaluated at steady state conditions.

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| --- |
| import sympy as sp  sp.init\_printing()  v,u = sp.symbols(['v','u'])  Fp,rho,Cd,A,m = sp.symbols(['Fp','rho','Cd','A','m'])  eqn = Fp\*u/m - rho\*A\*Cd\*v\*\*2/(2\*m)  print(sp.diff(eqn, v))  print(sp.diff(eqn, u)) |

|  |
| --- |
| def f(m, Fp, ρ, A, Cd, u, v):  return (1 / m) \* (Fp \* u - 0.5 \* ρ \* A \* Cd \* (v \*\* 2))  ρ = 1.225  A = 5  Cd = 0.24  m = 700  Fp = 30  f(700, 30, 1.225, 5, 0.24, 40, 40.406) |

**Exercise 2 – Thermocouple**

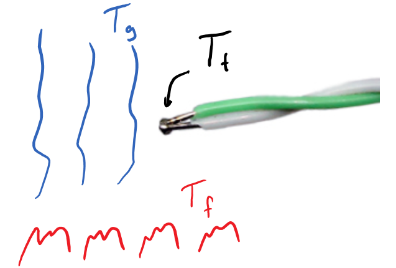
Linearize an [energy balance](http://apmonitor.com/pdc/index.php/Main/PhysicsBasedModels) that describes the temperature of a thermocouple bead Tt given a change in the surrounding gas temperature Tg. Include the flame temperature Tf as a variable in the linearization. Simulate in both the linear and nonlinear versions of the model:

* A cyclical gas temperature change
* A step change in flame temperature

Some advanced combustion systems operate in a pulsed mode where an acoustic wave resonates in the combustor in order to enhance heat and mass transfer. The frequency of the wave is approximately 100Hz (cycles/sec). As a result, the temperature at any given spatial location in the combustor fluctuates with time. The fluctuations can be approximated as a sine wave with an amplitude of 75 ˚C for the system of interest. **The objective is to measure the fluctuating temperature at a given point in the reactor with use of a thermocouple.**

Heat is transferred to the thermocouple by convection and radiation.

* Heat transfer by radiation is approximated by q=Aεσ(T4f−T4t) where A is the surface area, ε is the emissivity, and σ is the Stefan-Boltzmann constant (Wm2K4). The thermocouple bead is approximated as a sphere (e.g. ignore conduction along connecting wires).



Use the following values with subscript t as the thermocouple, subscript f as the flame, ss indicating steady-state, h as the heat transfer coefficient, cp as the heat capacity, ρ as the density, and dt as the diameter of the thermocouple bead.

h=2800Wm2K

ρt=20g/cm3

σ=5.67e−8Wm2K4

ϵ=0.8

Tt,ss=1500K

Tf=1500K

cp,t=0.4JgK

dt=0.01cm

Simulate both for the cycling gas temperature and discuss whether the linearized model is a suitable approximation of the nonlinear model. What changes could be made to the thermocouple to improve the ability to measure gas temperature?

**Solution**

Where:

: Heat lost by convection, in Watts. Newton’s law of cooling.

: Heat lost by radiative heat transfer, in Watts. Grey body model.

and:

: Thermocouple’s diameter

: Thermocouple’s surface area

: Thermocouple’s volume

: Thermocouple’s density

: Specific heat capacity

: Heat transfer coefficient

: Flame’s temperature

: Thermocouple’s temperature in steady state

: Emissivity factor [no units]

: Stefan–Boltzmann constant (5.67×10−8)

1. **Find the steady-state point**

* If steady state conditions, then